Exercise 8A

1 a $\int_2^5 x^3 dx = \left[\frac{x^4}{4}\right]_2^5$

$= \left(\frac{5^4}{4}\right) - \left(\frac{2^4}{4}\right)$

$= \frac{609}{4}$

$= 152\frac{1}{4}$

b $\int_1^3 x^4 dx = \left[\frac{x^5}{5}\right]_1^3$

$= \left(\frac{3^5}{5}\right) - \left(\frac{1^5}{5}\right)$

$= \frac{242}{5}$

$= 48\frac{2}{5}$

c $\int_0^4 \sqrt{x} dx = \int_0^4 x^{\frac{1}{2}} dx$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

$= \left[\frac{2x^{\frac{3}{2}}}{3}\right]_0^4$

$= \left(\frac{2(4)^{\frac{3}{2}}}{3}\right) - \left(\frac{2(0)^{\frac{3}{2}}}{3}\right)$

$= \frac{16}{3}$

$= 5\frac{1}{3}$

d $\int_1^3 \frac{3}{x^2} dx = \int_1^3 3x^{-2} dx$

$= \left[\frac{3x^{-1}}{-1}\right]_1^3$

$= \left[-\frac{3}{x}\right]_1^3$

$= \left(-\frac{3}{3}\right) - \left(-\frac{3}{1}\right)$

$= 2$

2 a $\int_1^2 \left(\frac{2}{x^3} + 3x\right) dx = \int_1^2 \left(2x^{-3} + 3x\right) dx$

$= \left(\frac{2x^{-2}}{-2} + \frac{3x^2}{2}\right)_1^2$

$= \left(-x^{-2} + \frac{3}{2}x^2\right)_1^2$

$= \left(-\frac{1}{4} + \frac{3}{2} \times 4\right) - \left(-1 + \frac{3}{2}\right)$

$= \left(-\frac{1}{4} + 6\right) - \frac{1}{2}$

$= 5\frac{1}{4}$

b $\int_0^2 \left(2x^3 - 4x + 5\right) dx = \left(\frac{2x^4}{4} - \frac{4x^2}{2} + 5x\right)_0^2$

$= \left(\frac{x^4}{2} - 2x^2 + 5x\right)_0^2$

$= \left(\frac{16}{2} - 2 \times 4 + 10\right) - (0)$

$= 8 - 8 + 10$

$= 10$

c $\int_4^9 \left(\sqrt{x} - \frac{6}{x^2}\right) dx = \int_4^9 \left(x^{\frac{1}{2}} - 6x^{-2}\right) dx$

$= \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{6x^{-1}}{-1}\right)_4^9 dx$

$= \left(\frac{2}{3}x^{\frac{3}{2}} + 6x^{-1}\right)_4^9$

$= \left(\frac{2}{3} \times 3^3 + \frac{2}{3}\right) - \left(\frac{2}{3} \times 2^3 + \frac{3}{2}\right)$

$= 18 + \frac{2}{3} - \frac{16}{3} - \frac{3}{2}$

$= 16\frac{1}{2} - \frac{14}{3}$

$= 11\frac{5}{6}$

2 d $\int_1^8 \left(x^{-\frac{1}{3}} + 2x - 1\right) dx = \left(\frac{x^{\frac{2}{3}}}{\frac{2}{3}} + \frac{2x^2}{2} - x\right)_1^8$

$= \left(\frac{3}{2}x^{\frac{2}{3}} + x^2 - x\right)_1^8$

$= \left(\frac{3}{2} \times 2^2 + 64 - 8\right) - \left(\frac{3}{2} + 1 - 1\right)$

$= 62 - \frac{3}{2}$

$= 60\frac{1}{2}$

3 a $\int_1^3 \left(\frac{x^3 + 2x^2}{x}\right) dx$

$= \int_1^3 \left(x^2 + 2x\right) dx$

$= \left(\frac{x^3}{3} + x^2\right)_1^3$

$= \left(\frac{27}{3} + 9\right) - \left(\frac{1}{3} + 1\right)$

$= 18 - \frac{4}{3}$

$= 16\frac{2}{3}$

b $\int_3^6 \left(x - \frac{3}{x}\right)^2 dx = \int_3^6 \left(x^2 - 6 + \frac{9}{x^2}\right) dx$

$= \int_3^6 \left(x^2 - 6 + 9x^{-2}\right) dx$

$= \left(\frac{x^3}{3} - 6x + \frac{9x^{-1}}{-1}\right)_3^6$

$= \left(\frac{x^3}{3} - 6x - 9x^{-1}\right)_3^6$

$= \left(\frac{216}{3} - 36 - \frac{9}{6}\right) - \left(\frac{27}{3} - 18 - \frac{9}{3}\right)$

$= 72 - 36 - \frac{3}{2} - 9 + 18 + 3$

$= 48 - \frac{3}{2}$

$= 46\frac{1}{2}$

c $\int_0^1 x^2\left(\sqrt{x} + \frac{1}{x}\right) dx = \int_0^1 \left(x^{\frac{5}{2}} + x\right) dx$

$= \left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{x^2}{2}\right)_0^1$

$= \left(\frac{2}{7} + \frac{1}{2}\right) - (0)$

$= \frac{11}{14}$

3 d $\int_1^4 \left(\frac{2 + \sqrt{x}}{x^2}\right) dx = \int_1^4 \left(\frac{2}{x^2} + \frac{1}{x^{\frac{3}{2}}}\right) dx$

$= \int_1^4 \left(2x^{-2} + x^{-\frac{3}{2}}\right) dx$

$= \left(\frac{2x^{-1}}{-1} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}\right)_1^4$

$= \left(-2x^{-1} - 2x^{-\frac{1}{2}}\right)_1^4$

$= \left(-\frac{2}{4} - \frac{2}{2}\right) - (-2 - 2)$

$= -1\frac{1}{2} + 4$

$= 2\frac{1}{2}$

4 $\int_1^4 (6\sqrt{x} - A)\, dx = A^2$

$\int_1^4 (6x^{\frac{1}{2}} - A)\, dx = A^2$

$\left[\frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - Ax\right]_1^4 = A^2$

$\left[4x^{\frac{3}{2}} - Ax\right]_1^4 = A^2$

$\left(4(4)^{\frac{3}{2}} - A(4)\right) - \left(4(1)^{\frac{3}{2}} - A(1)\right) = A^2$

$(32 - 4A) - (4 - A) = A^2$

$28 - 3A = A^2$

$A^2 + 3A - 28 = 0$

$(A + 7)(A - 4) = 0$

$A = -7$ or $A = 4$

5 $\int_1^9 (2x - 3\sqrt{x})\,dx = \int_1^9 (2x - 3x^{\frac{1}{2}})\,dx$

$$= \left[\frac{2x^2}{2} - \frac{3x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^9$$

$$= \left[x^2 - 2x^{\frac{3}{2}}\right]_1^9$$

$$\int_1^9 (2x - 3\sqrt{x})\,dx = \left(9^2 - 2(9)^{\frac{3}{2}}\right) - \left(1^2 - 2(1)^{\frac{3}{2}}\right)$$

$$= (81 - 54) - (1 - 2)$$

$$= 28$$

6 $\int_4^{12} \left(\frac{2}{\sqrt{x}}\right)dx = \int_4^{12} (2x^{-\frac{1}{2}})\,dx$

$$= \left[\frac{2x^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^{12}$$

$$= \left[4x^{\frac{1}{2}}\right]_4^{12}$$

$$= \left(4(12)^{\frac{1}{2}}\right) - \left(4(4)^{\frac{1}{2}}\right)$$

$$= 4\sqrt{12} - 8$$

$$= 4\sqrt{4 \times 3} - 8$$

$$= -8 + 8\sqrt{3}$$

7 $\int_1^k \frac{1}{\sqrt{x}}\,dx = 3$

$$\int_1^k x^{-\frac{1}{2}}\,dx = 3$$

$$\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^k = 3$$

$$\left[2\sqrt{x}\right]_1^k = 3$$

$$2\sqrt{k} - 2\sqrt{1} = 3$$

$$2\sqrt{k} = 5$$

$$\sqrt{k} = \tfrac{5}{2}$$

$$k = \tfrac{25}{4}$$

8 $s = \int_0^{10} (20 + 5t)\,dt$

$$= \left[20t + \frac{5t^2}{2}\right]_0^{10}$$

$$= \left(20(10) + \frac{5(10)^2}{2}\right) - \left(20(0) + \frac{5(0)^2}{2}\right)$$

$$= 450 \text{ m}$$

Challenge

$$\int_k^{3k} \frac{3x+2}{8}\,dx = 7$$

$$\int_k^{3k} \left(\frac{3x}{8} + \frac{1}{4}\right)dx = 7$$

$$\left[\frac{1}{2}\frac{3x^2}{8} + \frac{x}{4}\right]_k^{3k} = 7$$

$$\left[\frac{3x^2}{16} + \frac{x}{4}\right]_k^{3k} = 7$$

$$\left(\frac{3(3k)^2}{16} + \frac{(3k)}{4}\right) - \left(\frac{3k^2}{16} + \frac{k}{4}\right) = 7$$

$$\left(\frac{27k^2}{16} + \frac{3k}{4}\right) - \left(\frac{3k^2}{16} + \frac{k}{4}\right) = 7$$

$$\frac{24k^2}{16} + \frac{k}{2} = 7$$

$$24k^2 + 8k - 112 = 0$$

$$3k^2 + k - 14 = 0$$

$$(3k + 7)(k - 2) = 0$$

$$k = -\tfrac{7}{3} \text{ or } k = 2$$

As $k > 0$, $k = 2$